

Chirality in the Early Universe

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Abstract

The early big bang is an alphabet soup of quarks, W bosons, gluons, and other exotic particles and flavors. In the usual scenario, there is no place for the pion. It dissociates in the alphabet soup of the early universe. I will show that this scenario is naive. The thermal vacuum is a far more complex state, and the pion remains a Nambu-Goldstone particle at high T , and will not dissociate. It propagates at the speed of light but with a halo.

1 Introduction

In the usual folklore, chiral symmetry is restored in the early universe, and the pion is no longer a Nambu-Goldstone boson. The pion dissociates into massless quark and antiquark pairs.

An important part of the usual folklore revolves around the order parameter $\langle \bar{\psi} \psi \rangle$. At zero temperature, $\langle \bar{\psi} \psi \rangle$ is non-vanishing. The well-known theorem says that therefore the ground state is not chirally invariant. A corollary of this theorem is that for a chiral symmetric theory, there must then exist a massless Nambu-Goldstone particle, which we know as the pion. The fact that the physical pion we observe is not quite massless may be attributed to the presence of electroweak breaking in the Standard Model giving rise to a small primordial mass of the quarks.

What happens at high T ?

Studies have shown that $\langle \bar{\psi} \psi \rangle$ actually vanishes above a certain critical temperature, T_c . By analogy with the ferromagnetic system, the usual conclusion is drawn that a vanishing $\langle \bar{\psi} \psi \rangle$ indicates a chiral symmetry of the high temperature vacuum. *This conclusion is false, as I will show in an example. (See eq.(3) below)*

However, I hasten to add, this does not mean that there was no phase transition taking place at T_c . On the contrary, there is a very interesting new phase transition taking place. It is a morphosis of the

old zero temperature chirality. The original NJL vacuum undergoes an interesting *new phase transformation* such that $\langle \bar{\psi} \psi \rangle$ vanishes, but the vacuum continues to break our zero temperature chirality.

Above T_c , a new chiral symmetry takes over. The pion remains a Nambu-Goldstone boson, and actually acquires a halo while propagating through the early universe.

2 High Temperature Effective Action

At high temperatures, lattice work as well as continuum field theory calculations show that the effective action indeed exhibits a manifest chiral symmetry. In thermal field theory, there is the famous Braaten-Pisarski Frenkel-Taylor-Wong (BPFTW) action [1] that describes the propagation of a QCD fermion through a hot medium ($T'^2 \equiv \frac{g^2}{3}T^2$, while the angular brackets denote an average over the orientation \hat{n})

$$\mathcal{L}_{\text{eff}} = -\bar{\psi}\gamma_\mu\partial^\mu\psi - \frac{T'^2}{2}\bar{\psi}\left\langle\frac{\gamma_o - \vec{\gamma}\cdot\hat{n}}{D_o + \hat{n}\cdot\vec{D}}\right\rangle\psi \quad (1)$$

and we see the global chiral symmetry of the action. But the *nonlocality* of the action implies that the Noether charge for this new chirality is not the same as the usual zero temperature chirality.

The fermion propagator [2] that results from this action shows a pseudo-Lorentz invariant particle pole of mass T' (the so-called thermal mass) [3]. But, in addition, there is a pair of conjugate *spacelike* plasmon cuts in the p_o -plane that run just above and below the real axis [4], from $p_o = -p$ to $p_o = p$. The cuts are associated with the logarithms that result from the angular average in eq.(1). Along the real p_o axis, the propagator function has been chosen real. As a result, for $t > 0$, say, the propagator function takes the form

$$\begin{aligned} \langle T(\psi(x)\bar{\psi}(0)) \rangle_\beta = & \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \left\{ Z_p \frac{-i\vec{\gamma}\cdot\vec{p} + i\gamma_o\omega}{2\omega} e^{-i\omega t} \right. \\ & \left. - \frac{T'^2}{8} \int_{-p}^p \frac{dp'_o}{p^3} \frac{i\vec{\gamma}\cdot\vec{p}p'_o - i\gamma_o p^2}{p^2 - p_o'^2 + T'^2} e^{-ip'_o t} + O(T'^4) \right\} \quad (2) \end{aligned}$$

Note that the spinor structure of the massive particle pole term has the (unusual) feature of being manifestly chiral invariant. Here Z_p is a wave function renormalization constant [5].

In a recent study of the spacetime quantization of the BPFTW action [5], I have shown that the spacelike cuts dictate a new thermal vacuum of the type

$$|vac' \rangle = \prod_{p,s} \left(\cos \theta_p - i s \sin \theta_p a_{p,s}^\dagger b_{-p,s}^\dagger \right) |0 \rangle \quad (3)$$

The 90° phase here in the generalized NJL vacuum is the reason why $\langle \bar{\psi} \psi \rangle$ vanishes for $T \geq T_c$.

The quantization of a nonlocal action is of course a technical matter. Suffice it here to say that the quantization has been formulated in terms of auxiliary fields so that the resulting action is local. In this context, the pseudo-Lorentz particle pole is described in terms of the massive canonical Dirac field, Ψ , and the spacelike cuts are associated with the auxiliary fields, which are functions of Ψ . This formulation allows for a systematic expansion of the ψ field in terms of the massive canonical Dirac field, Ψ . Let the $t = 0$ expansion for the original massless ψ field read

$$\psi(\vec{x}, 0) = \frac{1}{\sqrt{V}} \sum_p e^{i\vec{p} \cdot \vec{x}} \left\{ \begin{pmatrix} \chi_{p,L} a_{p,L} \\ \chi_{p,R} b_{-p,R}^\dagger \end{pmatrix} + \begin{pmatrix} \chi_{p,R} a_{p,R} \\ -\chi_{p,L} b_{-p,L}^\dagger \end{pmatrix} \right\} \quad (4)$$

with a corresponding canonical expansion for the massive Ψ , then we find

$$a_{p,s} = A_{p,s} - i s \frac{T'}{2p} B_{-p,s}^\dagger + O(T'^2) \quad (5)$$

$$b_{p,s} = B_{p,s} + i s \frac{T'}{2p} A_{-p,s}^\dagger + O(T'^2) \quad (6)$$

The $O(T')$ terms in the Bogoliubov transformation imply the new thermal vacuum of eq.(3).

The chiral charge at high T is given by

$$Q_5^\beta = -\frac{1}{2} \sum_{p,s} s \left(A_{p,s}^\dagger A_{p,s} + B_{-p,s}^\dagger B_{p,s} \right) \quad (7)$$

so that it annihilates the new thermal vacuum, in direct contrast with the $T = 0$ Noether charge

$$Q_5 = -\frac{1}{2} \sum_{p,s} s \left(a_{p,s}^\dagger a_{p,s} + b_{-p,s}^\dagger b_{-p,s} \right) \quad (8)$$

which clearly fails to annihilate the vacuum at high T .

3 $\langle \bar{\psi} \psi \rangle$ is an Incomplete Order Parameter

The traditional order parameter $\langle \bar{\psi} \psi \rangle$ cannot by itself give a full description of the nature of chiral symmetry breaking. The operator, $\bar{\psi} \psi$, belongs to a non-Abelian chirality algebra, $SU(2N_f)_p \otimes SU(2N_f)_p$. This algebra is not to be confused with the $U(1)_V \otimes SU(N_f)_L \otimes SU(N_f)_R$ algebra that is a symmetry of the fundamental massless Lagrangian. This symmetry is broken spontaneously by the vacuum to the surviving vector symmetry $U(1)_V \otimes SU(N_f)_V$.

The $SU(2N_f)_p \otimes SU(2N_f)_p$ chirality algebra we refer to here is instead a spectrum generating algebra, so that in general elements of the algebra do not commute with the Hamiltonian. They are nevertheless useful in classifying the properties of the dynamical vacuum that result.

The original chiral broken NJL ground state may be written as an X_2 rotation of the usual Fock vacuum

$$|vac\rangle = \prod_p e^{iX_{2p}\theta_p} |0\rangle \quad (9)$$

where X_{2p} is an element of the algebra, given in terms of the massless quark operators by

$$X_{2p} = i \sum_s \frac{s}{2} \left(a_{p,s}^\dagger b_{-p,s}^\dagger - b_{-p,s} a_{p,s} \right) \quad (10)$$

while the new thermal vacuum eq.(3) is generated by a different element, Y_{1p} ,

$$Y_{1p} = - \sum_s \frac{s}{2} \left(a_{p,s}^\dagger b_{-p,s}^\dagger + b_{-p,s} a_{p,s} \right) \quad (11)$$

It is interesting to note that the usual order parameter is related to this element through the volume integral $\frac{1}{2} \int d^3x \bar{\psi}(\vec{x}) \psi(\vec{x}) = - \sum_p Y_{1p}$. The remaining elements of the algebra exhaust the bilinears that may be formed from the massless quark operators

$$X_{3p} = - \sum_s \frac{s}{2} \left(a_{p,s}^\dagger a_{p,s} + b_{-p,s}^\dagger b_{-p,s} \right) \quad (12)$$

$$X_{1p} = \sum_s \frac{1}{2} \left(a_{p,s}^\dagger b_{-p,s}^\dagger + b_{-p,s} a_{p,s} \right) \quad (13)$$

$$Y_{1p} = \sum_s \frac{s}{2} \left(a_{p,s}^\dagger b_{-p,s}^\dagger + b_{-p,s} a_{p,s} \right) \quad (14)$$

$$Y_{3p} = \sum_s \frac{1}{2} \left(a_{p,s}^\dagger a_{p,s} - b_{-p,s} b_{-p,s}^\dagger \right) \quad (15)$$

The X operators generate the $SU(2)$ algebra, with X_{3p} easily recognizable as the Fourier component of the usual chirality charge, $Q_5 = \sum_p X_{3p}$. While X_3, Y_1 are related to local operators in coordinate space, the other elements are related to necessarily nonlocal operators. Our results here suggest the study of a new class of these nonlocal order parameters that involve a time integration that projects away the usual timelike spectrum of the operator ψ , and probes directly the properties of the spacelike cut.

4 Pion halo in the Sky

The pion we know at zero temperature is not massless, but has a mass of 135 MeV . This is because of electroweak breakdown, giving rise to a primordial quark mass at the tree level. At very high T , when electroweak symmetry is restored, we have the interesting new possibility that the pion will fully manifest its Nambu-Goldstone nature and remain physically massless. [6]

The pion is described by an interpolating field operator, $\pi^a \sim i\bar{\psi}\gamma_5 T^a \psi$, which does not know about temperature. It is the vacuum that depends on T . The state vector for a zero momentum pion at high T may be obtained from the thermal vacuum by the action

$$Q_5^a |vac' \rangle \propto |\pi^a(\vec{p}=0)\rangle \quad (16)$$

This pion now has the property that even though it is massless, it can acquire a *screening mass* proportional to T . This is the pion mass that has been measured on the lattice at high T .

As a result, the pion propagates in the early universe with a halo. The retarded function for the pion shows that the signal propagates along the light cone, with an additional exponentially damped component coming from the past history of the source.

$$D_{\text{ret}}(\vec{x}, t) = \theta(-t) \left\{ \delta(t^2 - r^2) + \frac{T'}{r} \theta(t^2 - r^2) \left[e^{-T'|t-r|} + e^{-T'|t+r|} \right] \right\} \quad (17)$$

The screening mass leads to an accompanying modulator signal that ‘hugs’ the light cone, with a screening length $\propto 1/T$.

What are the cosmological consequences of a pion in the alphabet soup of the early universe?

In the usual scenario, the pion after chiral restoration will have acquired mass $\propto T$, and will quickly dissociate into constituent quark-antiquark pair. According to our new understanding, however, the Nambu-Goldstone theorem forces the pion to remain a strictly massless bound state at high T , and so the pion will contribute to the partition function of the early universe.

Fortunately, the pion does not contribute so many degrees of freedom as to upset the usual picture of the cooling of the universe. But I leave it to experts to help figure out the subtle changes there must surely be in the phase transitions of the early universe.

In the beginning there was light, and quarks, and gluons, to which we must now add the pions with halo.

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